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ANALYSE THE OPTIMAL ERROR AND SMOOTHNESS VALUE OF CHEBYSHEV WAVELET

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ABSTRACT

In this Chapter, we obtained Wavelet error analysis of optimal control in nonlinear differential equation by apply the Chebyshev Wavelet and to analyze the Residual of Wavelet Smoothness $N_2(e)$. This method transforms the nonlinear systems of differential equation to nonlinear systems of statistical equations. The convergences of the numerical method are given and their applicability is illustrated with some examples.

KEYWORDS: Wavelet, Chebyshev Polynomial, Shifted Chebyshev Polynomial.

INTRODUCTION

The wavelet methods are very effective and efficient tool for solving mathematical problems. It has many useful properties. Wavelets can be used for algebraic operations in the system of equations obtained which lead to better condition number of the resulting system. In wavelet based methods, there are two important ways of improving the approximation of the solutions: Increasing the order of the wavelet family and the increasing the resolution level of the wavelet. There is a growing interest in using various wavelets to study problems, of greater computational complexity. Among the wavelet transform families the Haar, Legendre and Chebyshev wavelets deserve much attention. The basic idea of Chebyshev wavelet method (CWM) is to convert the differential equations to a system of algebraic equations by the operational matrices of integral or derivative. The main advantage of these methods lies in their accuracy for a given number of unknowns. In contrast, finite difference and finite-element methods yield only algebraic convergence rates. In [9], a method for numerical solution of nonlinear equations utilizing positive definite functions is proposed by Alipanah and Dehghan. [7] Introduced a numerical method for solving nonlinear. In [1], a Chebyshev finite difference method has been proposed in order to solve linear and nonlinear second-order Fredholm integro-differential equations. First time, Chebyshev wavelet as another polynomial wavelet was discussed in [12, 11]. Using of chebyshev wavelet to solve integral equations continued in some other papers [9, 10]. Many researchers [3, 2, 4, 1, 13] derived the properties of chebyshev polynomial in wavelet fundamentals. The main purpose of this article using the properties of Chebyshev wavelet for solving Eq. (1) and Eq. (2) and obtain the error optimal value of Wavelet function and to analyze the residual analyze of NDE. The interval of the system lies between 0 < t < 1.

Shifted Chebyshev polynomials

The range [0, 1] is useful to the range of the interval [-1, 1]. Sometime the variable x is independent in [0, 1] that time variable x transform to s in [-1, 1]. The transformation defined as s=2x -1 or x=0.5(s+1). This transformation shifted to the chebyshev polynomial $T_n^*(x)$ of nth degree is given by $T_n(x)=T_n(s)=T_n(2x-1)$. Thus we have the polynomial is $T_0^*(x)=1, T_1^*(x)=2x-1$, $T_2^*(x)=8x^2-8x+1$ … In general recurrence relation of the $T_n^*(x)$ is $T_n^*(x)=$ 2(2x-1) $T_{n-1}^*(x)$ - $T_{n-2}^*(x)$, here $T_0^*(x)$ and $T_1^*(x)$ are initial conditions

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Properties of the Shifted chebyshev polynomial:

[5,8] are derived the recurrence formula of the chebyshev polynomials is

$$T^*_{n,r+1}(x) = 2\left(\frac{2x-n}{n}\right)T^*_{n,r}(x) - T^*_{n,r-1}(x),\ r = 1,2,...$$

Here r is the highest degree of polynomial

$$T^*_{n,r} = r\sum_{m=0}^{r}(-1)^{r-m}\frac{(r+m-1)!2^{2m}}{(r-m)!(2m)!nm}(xm),$$ where $T^*_{n,r}(0) = (-1)^r$ and $T^*_{n,r}(n) = 1$.

The orthogonal condition of the integral equation is $\int_0^n T^*_{n,i}(x)T^*_{n,j}(x)\,w(x)\,dx = h_k\ \mu_{ij}$, Where $w(x) = \frac{1}{\sqrt{nx - x^2}}$,

here $a_0(x) = 1\ and\ a_i(x) = n$ and $h_k(x) = a_i(x)\pi/2$

The k^th^ derivative of the r^th^ order shifted polynomial defined as: $T^{(k)^*}_{n,r} = r\sum_{m=s}^{r}(-1)^{r-m}\begin{pmatrix} m \\ k-1 \end{pmatrix}\frac{(r+m-1)!2^{2m}}{(r-m)!\,n^k}\,x^{m-k}$

Proposed Algorithm:

Consider the Nonlinear Differential Equation:

$f^{(P)}(x) = \sum_{i=1}^{m} p_{ij}(x, y_i, y_i^{(k)})$ ---------- (1) Where p = 1, 2…n and a < x < b

Additional condition of the NDE is $\sum_{i=1}^{m}\sum_{k=1}^{p}(C_{i\,k}\,y_i^{(k_i-1)}(x_1) + D_{i\,k}\,y_i^{(k_i-1)}(x_2) = \gamma$ ----------- (2) where $f^{(p)}(x)$ is an analytic function, C and D are constant polynomials. In order to solving equation (1) by using comparison method, we obtained the approximate $y_N^i(x)$ defined as: $y_N^i(x) = \sum_{r=0}^{N} b_r^i T_r^*(x)$ -------- (3), where b_r^i, $r = 1,2....N$, N > 0, r is the unknown shifted chebyshev coefficients.

From (1) and (3) we have $\sum_{i=1}^{m} P_{ij}(x, \sum_{r=0}^{N} b_r^i\,T_r^*(x), ...\sum_{r=k_i}^{N} b_r^i(T_r^*)^{k_i}) = f_k^p(x)$ ---------- (4)

Substituting equation number (2) in equation (3), we obtained the k^th^ polynomial equation as follows:

$$\sum_{i=1}^{m}\sum_{k=1}^{p}\left(C_{ikr}\sum_{r=k_i-1}^{N} b_r^i\,T_r^{*(k_i-1)}(x_1) + D_{ikr}\sum_{r=k_i-1}^{N} b_r^i\,T_r^{*(k_i-1)}(x_2)\right) = \gamma \text{ ------- (5)}$$

From Equation (4 and 5), we provide the $m \times (N+1)$ nonlinear numerical equations. Solving equation (5) we find the unknown coefficients of b_i^r. Therefore, we using ORIGIN 0.6 in equation (3), easily obtain the approximate value of N various solution and analyze the frequency of the chebyshev wavelet.

Proposed Algorithm of Truncated Error Analysis:

Assume that y(x) is a function on [0, 1], $\phi_N(x)$ is the interpolating polynomial to y at x_i, i= 1, 2, 3….n are the chebyshev interpolating points, then we've

$$y(x) - \phi_N(x) = \left(\prod_{i=0}^{N}\frac{y^{(N+1)}\gamma}{(N+1)!} \times (x - x_i)\right)$$

From [12, 10, 6], we derived the N^th^ polynomial as follows: $\left|y(x)-y_N(x)\right| \le \frac{1}{2^{N+1}}\left\|y^{(N+1)}(x)\right\|$ --- (6)

Theorem: 1

Consider the function f(x) is continuous and satisfying the condition on [-1, 1], then the series expansion of chebyshev polynomial is uniformly convergent.

Proof: Let the function f(x) implies the same condition for $f(\theta)=f(\cos\theta)$.

$$f(\theta+\delta)-f(\theta) = \left|f(\cos(\theta+\delta)-f(\cos\theta)\right|$$
$$\le(\varpi(\cos(\theta+\delta)-\cos(\theta))$$
$$\le\varpi(\delta)$$

Since, it is easily shown that $\left|(\cos(\theta+\delta)-(\cos\theta)\right|\le\delta$, $\varpi(\delta)$ is an increasing function of δ

Theorem: 2

Suppose the function (N+1) times continuously differential on [0,1] and the exact solution of the chebyshev polynomial is $y_N^i(x)=\sum_{r=0}^{N} b_r^i T_r^*(x)$. The approximate solution is obtained

$$\left\|y(x)-y_N(x)\right\|^2 \le \frac{1}{2^{N+1}}\left\|y^{N+1}(x)\right\|+\sqrt{\frac{\pi}{8}}\left\|\eta-\overline{\eta}\right\|^2$$

Proof: Let $y_N(x)$ is Nth order degree off polynomial and $y_N(x)$ is the approximate solution of $y(x)$. Therefore, we can write the formula

$$\left\|y(x)-y_N(x)\right\|^2 \le \left\|y(x)-\overline{y_N}(x)\right\|+\left\|\overline{y_N}(x)-y(x)\right\| \quad \text{------- (7)}$$

Using equation (6) in (7) , we obtain

$$\left\|y(x)-y_N(x)\right\|^2 = \left(\int_0^1 \left|y(x)-y_N(x)\right|^2 dx\right)^{1/2}$$

$$\le \int_0^1 \frac{1}{2^{N+1}}\left(\frac{1}{(N+1)!}\left\|y^{(N+1)}(x)\right\|^2 dx\right)^{1/2} \quad \le \in\left(\frac{1}{2^{N+1}(N+1)!}\left\|y^{(N+1)}(x)\right\|\right) \text{------- (8)}$$

$$\left\|\overline{y_N}(x)-y_N(x)\right\|^2 = \left(\int_0^1\left[\sum_{r=0}^{N}(b_r-\overline{b_r})\,T_r^*(x)\right]^2 dx\right)^{1/2} \quad \le \int_0^1\left[\sum_{r=0}^{N}(b_r-\overline{b_r})^2\left[\sum_{r=0}^{N}\left|T_r^*(x)\right|^2\right]dx\right]^{1/2}$$

$$\le\left\|\eta-\overline{\eta}\right\|\int_0^1\sum_{r=0}^{N}\left|T_r^*(x)\right|dx \quad \le\left\|\eta-\overline{\eta}\right\|\sqrt{\pi/8} \text{ --------------- (9)}$$

Therefore, adding the equation 8+9, we obtained the approximate solution , From equation (1), the function $y_N^i(x)$, i=0, 1, 2….m, Using the properties of Chebyshev wavelets the truncated error estimated as

$$\left|f^{(P)}(x)-\sum_{i=1}^{m} p_{ij}(x,y_i,y_i^{(k)})\right| \cong 0 \quad \text{---------- (10)}$$

Therefore $E_N(x)=f^{(p)}(x)-\sum_{i=1}^{m} p_{ij}(x,y_i,y_i^{(k)})$

Numerical Example:

Consider the NLDE $y_1^{''}(x)+xy_1^{'}(x)+\cos(x)y_2^1(x)=f_1\ (x) \quad 0\le x\le 1$ and

$y_2^{''}(x)+xy_1^{'}(x)+xy_1^2(x)=f_2\ (x) \quad 0\le x\le 1$, the boundary condition is

$$y_1(0)=y_1(i)=y_2(0)=y_2(1)=0$$

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Where $f_1(x)=\sin(x)+\left(x^2-5x+6\right)\cos(x)+(5-2x)\cos(\pi x)$ and

$f_2(x)=-6+x\sin(x)+x(x-5)^2\sin^2(x)+(x^2-5x)\cos x$

The approximate solutions are $y_1(x)=x^2-5x$ and $y_2(x)=(x-5)\sin(x)$, Using algorithm, we have N=4

$$\sum_{r=2}^{4} b_r^1 T_r^{2*}(x_m)+\sum_{r=1}^{4} x_m b_r^1 T_r^*(x_m)+\sum_{r=1}^{4} 2x_m b_r^2 T_r^*(x_m)+x_m\left(\sum_{r=0}^{4} b_r^1 T_r^{2*}\right)^2 = f_1(x_q) \text{ -------- (11)}$$

$$\sum_{r=2}^{4} b_r^2 T_r^{2*}(x_m)+x_m\sum_{r=1}^{4} b_r^1 T_r^*(x_m)+\sum_{r=0}^{4} x_m b_r^1 T_r^*(x_m)=f_2(x_q) \text{ ------------------------------- (12)}$$

Where q= 0,1,2,3 are roots of the shifted Chebyshev polynomial $T_4^*(x)$, necessary condition as follows:

$$y_1^4(0)=\sum_{q=0}^{3}(-1)^q b_r^1=0 \text{ -------- (13)},\quad y_1^4(1)=\sum_{q=0}^{3} b_r^1=0 \text{ --------------- (14)}$$

$$y_2^4(0)=\sum_{q=0}^{3}(-1)^q b_r^2=0 \text{ ------- (15)},\quad y_2^4(1)=\sum_{q=0}^{3} b_r^2=0 \text{ --------------- (16)}$$

From equation (14 to16), we obtained the 8 nonlinear equations with the eight unknown coefficients. Solving the above equation and substituting the equation (3), we get the estimated solution of N=4

$$y_4^1(x)=x^2-5x,\ y_4^2(x)=0.6823x+x^2+0.4325x^2+0.4307x^4 \text{ ____________ (17)}$$

Using our proposed algorithm and iterative method (IM), the result on exact solution of $y_N^2(x)$ for N=4.,

Table: 1 Error Value Table

X	$y_4^2(x)$ (N=2)	$y_4(x)$ (N=1)	$N_e(2)$	IM	Error
0	0.12152	-.49	-0.9154	-.1.002	0.0866
0.1	0.12152	-0.49	-0.9154	-0.900	-0.0154
0.2	0.26364	-0.96	-0.5790	-0.893	0.314
0.3	0.42792	-1.41	-0.3685	-0.732	0.3635
0.4	0.61694	-1.84	-0.2097	-0.731	0.5213
0.5	0.83431	-2.25	-0.0786	-0.532	0.4534
0.6	1.08469	-2.64	0.0352	-0.319	0.3542
0.7	1.37377	-3.01	0.1378	-0.098	0.2358
0.8	1.70825	-3.36	0.2325	0.124	0.1085
0.9	2.09590	-3.69	0.3213	0.267	0.0543
1	2.54550	-4	0.4057	0.367	0.0387

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Fig (i) Analyze the Wavelet Smoothness of $N_e(2)$ ***and IM***

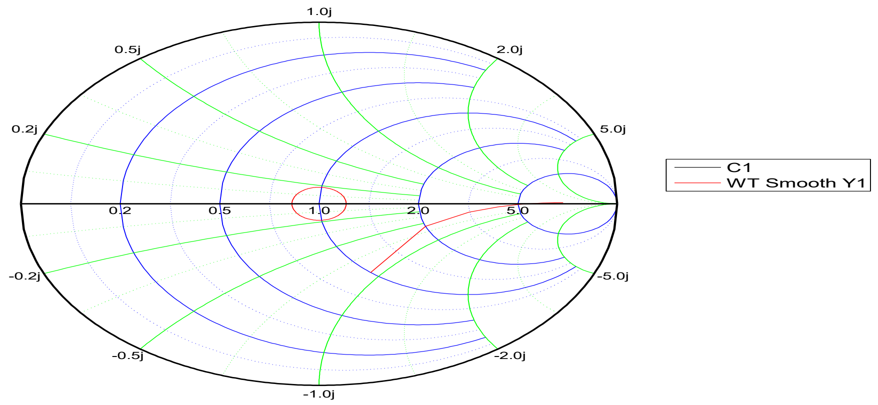

Fig (2) Analyze the Wavelet Denoised of $N_e(2)$ ***and IM***

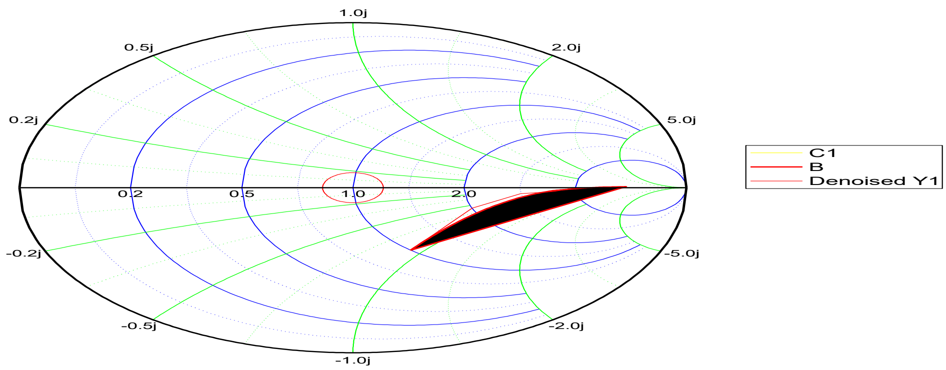

Fig (3) Analyze the Wavelet smoothness and Denoised of $N_e(2)$ ***and IM***

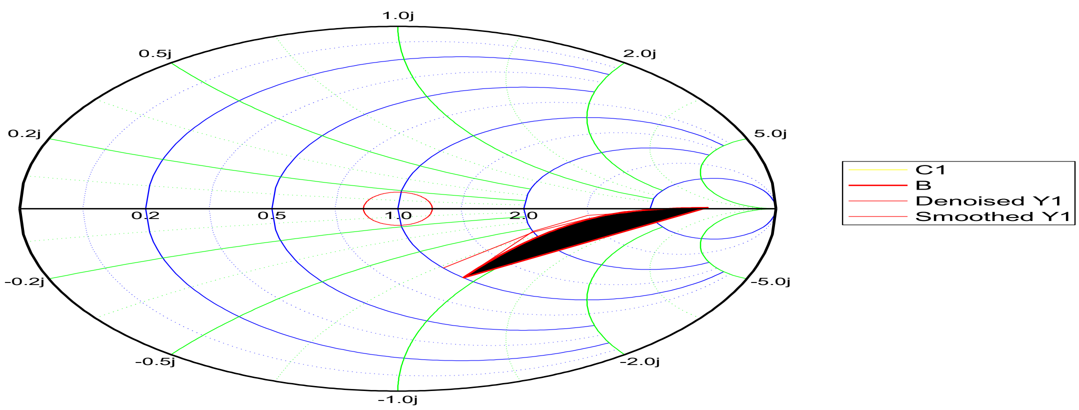

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Table .2: Standard Error Value of $N_2(e)$ and IM

		VALUE	SE
WT Smooth $N_2(e)$	Intercept	-1.10101	0.05055
	$N_2(e)$	2.62487	0.21112
	IM	-1.15466	0.18705

Table 3: DF of $N_2(e)$ and IM:

	WT Smooth $N_2(e)$
Number of Points	10
Number of Points	7
Degrees of Freedom	0.01293
Degrees of Freedom	0.98959

Table 4: Summary of the statistics:

Intercept	Intercept	$N_2(e)$	$N_2(e)$	IM	IM	Statistics
Value	Error	Value	Error	Value	Error	Adj. R-Square
-1.10101	0.05055	2.62487	0.21112	-1.15466	0.18705	0.98959

Table 5: Using Anova Value:

		DF	Sum of Squares	Mean Square	F Value	Prob>F
WT Smooth $N_2(e)$	Model	2	1.58455	0.79227	428.8759	4.77225E-8
	Model	7	0.01293	0.00185		
	Error	9	1.59748			

Table 6: Adjusted Residual of Polynomial Value:

	Value	SE
WT Smooth $N_2(e)$	2.62487	0.21112
WT Smooth IM	-1.15466	0.18705

Fig 4: Residual vs Independent Variable:

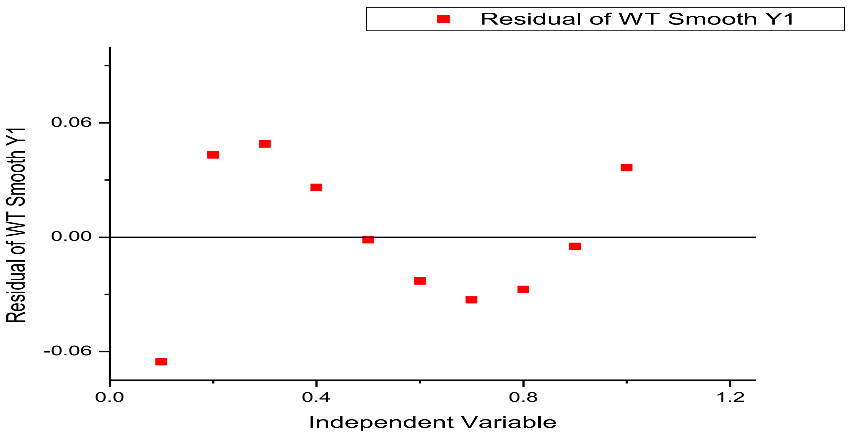

Table: 7 Calculated Value of $N_2(e)$

Independent Variable	Residual of WT Smooth $N_2(e)$
0.1	-0.0653354545454544
0.2	0.0432172727272728
0.3	0.0489631818181818
0.4	0.0261022727272727
0.5	-0.00136545454545467
0.6	-0.0230400000000002
0.7	-0.0328213636363637
0.8	-0.0274095454545455
0.9	-0.00480454545454562
1	0.0364936363636362

CONCLUSION

In this paper, proposed a new algorithm to obtained Wavelet error analysis of optimal control in nonlinear differential equation by apply the Chebyshev Wavelet and to analyze the Residual of Wavelet Smoothness and our proposed methodology is compared with the iterative method.. This method is very useful to quickly identify the error performance.

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